Auszug aus einer Elektronik Vorlesung von Dr. Wilfried Tenten

1.1.1 Introduction to Noise Theory

This section will describe

- thermal noise
- shot noise
- generation and recombination noise
- flicker noise

Within that section, the fundamentals will be laid to describe noise within circuits.

Thermal noise was predicted from the studies of Brownian motion [2] (Mind that the origin of noise theory by Leeuwenhoek [1] was lost over more than two centuries). It was J.B. Johnson of Bell Laboratories in 1927 and a theoretical analysis was provided by H. Nyquist in 1928. Because of their work, thermal noise is called Johnson or Nyquist noise [3]. The thermal noise is the energy “floor”, that means a rough floor of peaks observable in the frequency domain over a wide range.

Shot noise is produced within p-n junctions and is mostly not relevant in the field of MOS noise. This is based on the current flow, which is a drift of charge carriers through ohmic conduction and not by charge carriers crossing a potential barrier. [4]

Generation and recombination noise is produced by free charge carriers by a statistical variation of their time-constants. [5]

Flicker noise shows an indirect relationship between the frequency and the resulting equivalent input noise voltage. This noise is just relevant at low frequencies. [6]

Publications on noise most is out from the internet from WIKIPEDIA. From here, a lot of other references can be down-loaded.

1.1.1.1 Noise of Resistors

$$\rho(t)=2\cdot R\cdot(\pi\cdot k\cdot T)^2\left(\left(\frac{h}{2\cdot\pi^2\cdot k\cdot T\frac{1}{f}}\right)^2-\frac{1}{\sinh^2\left(\frac{2\cdot\pi^2\cdot k\cdot T\frac{1}{f}}{h}\right)}\right)$$

eq. 1

The power consumption of resistors generates thermal energy. This energy can be described as a random movement of particles. The auto-correlation-function (ACF) of a noise emitting resistor is shown in the next equation [7]:

This equation shows no dependence on supply voltage. The Fourier transform of this equation gives the spectrum of the noise power density ω(f), shown in eq. 2.3.

$$\omega(f)=4\cdot R\cdot k\cdot T+\frac{h\cdot f}{k\cdot T\left(e^{\frac{h\cdot f}{k\cdot T}}-1\right)}$$

eq. 2

h is the quantized Planck constant: 6.6 10^{-34} WS^2

k is the Boltzmann constant: 1.38 10^{-23} Ws/K

The explicit computation of the right hand term in eq. 2.3 inside the frequency range up to 1 GHz is much less than 1. Therefore, this equation can be simplified and using the Nyquist law, we can write it as:

$$\omega(f)\cdot\Delta f=4\cdot k\cdot T\cdot R\cdot\Delta f=\bar{v}^2$$

eq. 3

This is the well known formula for noise in resistors. You can use this formula as a resistor noise model in MATLAB SIMULKINK models.

1.1.1.2 Noise in MOS structures

As explained above, a MOS transistor can be discussed as a voltage controlled current source. Respectively to the limited resistance of the channel, which can be seen as r_{DS}, we also can abbreviate the MOS description as a voltage controlled resistance. Following our noise equations of the section before, we expect a noise floor like a resistance. The drain current versus the drain-to-source voltage was also shown (see section 1) and studying this plot, we never do see a straight line. So, we do not have an Ohmic characteristic. Following this non-linearity of the r_{DS}, we can see a descent of noise power coming from the lower frequencies to higher frequencies and at a specific region, the noise remains nearly constant. The thermal noise of a MOS structure is not easy to derive by pure applying the theory. Based on experience, we can derive a noise model by educated guessing and this equation shows a dependence of the g_m, the forward conductance.

$$\frac{\bar{v}^2_{nT}}{\Delta f}=\frac{4\cdot k\cdot T}{\frac{2}{3}\cdot gm}$$

$$\bar{v}^2_{nT}=\int_{f1}^{f2}\frac{\bar{v}^2_{nT}}{\Delta f}\,df=\frac{4\cdot k\cdot T}{\frac{2}{3}\cdot gm}\cdot(f2-f1)$$

eq. 4

It is interesting to have here a situation of something unknown of difficult to analyse with scientific procedures. Allow me to say, we have here a very typical situation for engineers. Make a comparison to the transistor model. A MOS or a bipolar transistor model can be derived by theory. The procedure is to examine the electrical fields and from there the terminal description can be derived. In case, you need a transistor model in a simulation of higher level of abstraction, e.g. MATLAB SIMULINK, then this procedure is not very wise to follow.

Here a model derived by educating guessing is probably the best way to come across solving your system. What is to do? You have to learn sharpening your eyes to the characteristics seen, eg. in laboratory experiments or by using a network analyser. From here, you must ask, which mathematical description may be applied to achieve the same or a very close characteristic. In our practices, you will learn this and you will see how to do it.

$$\frac{\bar{v}^2_{0n}}{\Delta f}=\bar{v}^2_n\cdot\left(\frac{G}{G+j\cdot\omega\cdot C}\right)$$

$$\bar{v}^2_{0n}=\int_0^\infty\left(\frac{\bar{v}^2_n}{\Delta f}\cdot\frac{G}{G^2-(\omega\cdot C)^2+j\cdot 2\cdot G\cdot\omega\cdot C}\right)d\frac{\omega}{2\cdot\pi}$$

$$\bar{v}^2_{0n}=\frac{4\cdot k\cdot T\cdot R\cdot G^2}{2\cdot\pi}\cdot\int_0^\infty\left(\frac{1}{G^2-(j\cdot\omega\cdot C)^2}\right)d\,\omega$$

eq. 5

On silicon, we have a lot of capacitances, most of them are parasitic. Transistors are at work, so we have a continuous change in channel or collector-emitter resistance. The noise produced by the parasitic capacitances connected to signal lines can be derived by taking the transistor conductances G in account.

$$\bar{v}^2_{0n}=\frac{4\cdot k\cdot T}{2\cdot\pi\cdot C}\cdot atan\left(\frac{j\cdot\omega\cdot C}{G}\right)$$

$$\bar{v}^2_{0n}=\frac{4\cdot k\cdot T}{2\cdot\pi\cdot C}\cdot\left(\frac{\pi}{2}-0\right)=\frac{K\cdot T}{C}$$

eq. 6

Equation 2.7 shows the famous “KT over C” noise.

Keep always in mind, a capacitance uni-solo has no noise, same an inductance!!!

$$f(x)=atan(x) \qquad f(z)=atan(z)$$

$$f(z)=\frac{1}{G^2\cdot\left(1+\left(\frac{j\cdot\omega\cdot C}{G}\right)^2\right)}$$

$$f(z)=\frac{1}{1+\left(\frac{j\cdot\omega\cdot C}{G}\right)^2} \quad = \quad atan\frac{j\cdot\omega\cdot C}{G}$$

$$\varphi=atan\frac{\alpha}{\beta}$$

eq. 7

The noise contribution is based on the combination of such an element with a branch of the circuit. Therefore we have to discuss any noise with capacitances and inductances as the result of a low- or high pass

Let me explain another consequence of this kind of noise. The integral term of eq. 2.6 will be rearranged:

From theses relations, we see that noise has an influence of the angle. This means that low frequency noise components may take influence of the system signal. In consequence can this means that we do see inaccuracies of our results expected and the origin of inaccuracy is noise! This is done in eq. 2.8.

Noise is ever present as I said before and can never been avoided. But as better the noise characteristics are known as better ideas we may develop to improve our circuit to withdraw noise. Later a figure named noise-shaping will be introduced. This noise-shaping takes such non-linearities in its procedure and finally we see that noise can be handled pretty similar as handling signals. And these equations build the basis on the idea of ***noise-shapers***.

1.1.1.3 Flicker noise in MOS Transistors

The origin of flicker noise is in the fluctuations of channel charges due to trapping by surface charges. Flicker noise decreases down to the constant thermal noise floor with an increase of frequency. This noise has a significant effect in MOS designs. Hence low noise designs should be optimized to the frequency dependent flicker noise first. McCreary gives the relation between the equivalent flicker noise voltage and the frequency as [7]:

A rearrangement of this equation with gm expressed from the transistor geometries gives:

$$\bar{v}^2_{nF}=\frac{K_F\cdot I_D}{C_{Ox}\cdot L^2\cdot g_m}\cdot\frac{1}{f}$$

eq. 8

W: Transistor width

L: Transistor length

K_F: flicker noise constant

C_{OX}: Oxide Capacitance

Noise optimization of circuits require a large area, this is what equation 2.10 tells. Just to produce a number: pick W/L = 10, as oxide thickness 70nm and a frequency of 1 KHz Then the noise power density is 95 nV per root Hertz. This “root Hertz” is the unit, when noise is named as a voltage. In our modern, ultra small active regions, the 1/f noise follows more the Lorentzian spectrum given as:

$$\frac{\bar{v}^2_{nF}}{\Delta f}=\frac{K_F}{2\cdot\mu\cdot C^2_{Ox}}\cdot\frac{1}{W\cdot L}\cdot\frac{1}{f}$$

$$S=\frac{\tau}{1+4\pi^2 f^2\tau^2}$$

eq. 10

This noise is included in all new network simulators: SPICE, SABER, SPECTRE etc. K.K. Kung et al [18, 19] presented that noise assume a trapping and de-trapping influence of charge carriers. At this place it is to note, that in modern CMOS processes the oxide thickness of poly-poly capacitance is extremely thin, around (10-20) nm and therefore we also see such effects, which may produce a space charge region -most likely unexpected by designers!.

This space charge region forces a CV characteristic of poly-poly capacitances!!

1.1.2 Noise of a transfer-gate

$$\bar{v}^2=\frac{k\cdot T}{C}\cdot\left(1-e^{-\frac{2\cdot t}{R\cdot C}}\right)$$

$$\bar{v}^2=\frac{k\cdot T}{C}$$

eq. 11

Most of the noise is induced by the thermal activity of charge carriers within the channel. The equivalent noise voltage must be expanded in switched capacitor circuits (SC) to include the time constant composed of the on-resistance of the transfer-gates and also to the inclusion of the load capacitance. The noise contribution can be written as shown with eq. 2.11.

1.1.3 Noise of an Inverter

$$\bar{e}_{out}^{\,2}=\bar{e}_{n1}^{\,2}\cdot\left(\frac{g_{gm1}}{g_{gm2}}\right)^2+\bar{e}_{n2}^{\,2}\quad for\,Vin=0$$

$$\bar{e}_{q}^{\,2}=\bar{e}_{n1}^{\,2}+\left(\frac{g_{gm1}}{g_{gm2}}\right)^2\cdot\bar{e}_{n2}^{\,2}=\frac{\bar{e}_{out}^{\,2}}{\left(\frac{g_{gm1}}{g_{gm2}}\right)^2}$$

Based on an inverter with an active load as shown in illustration 6, the noise evaluation can be done by inclusion of a noise source at each input. Such noise sources shall be connected in series with the signal. In fact we do have then a noise signal, which is modulated on the system signal. The term e means a noise source indexed by 1 and 2. In respect to illustration 6, “1” is referenced to the signal input transistor and “2” to the active load transistor. The index q stands for the common noise source.

$$r_{out}=\frac{1}{g_{DS1}+g_{DS2}}$$

$$\bar{e}_{out}^{\,2}=\left(g_{m1}\cdot r_{out}\right)^2\cdot\bar{e}_{n1}^{\,2}+\left(g_{m2}\cdot r_{out}\right)^2\cdot\bar{e}_{n2}^{\,2}$$

$$V_{out}=-V_{inp}\cdot\left(g_{m1}+g_{m2}\right)\cdot r_{out}$$

$$\bar{e}_{eq}^{\,2}=\left(\frac{g_{m1}}{g_{m1}+g_{m2}}\right)^2\cdot\bar{e}_{n1}^{\,2}+\left(\frac{g_{m2}}{g_{m1}+g_{m2}}\right)^2\cdot\bar{e}_{n2}^{\,2}$$

$$\bar{e}_{eq}^{\,2}=\bar{e}_{n1}^{\,2}\cdot\left(\frac{1+\left(\frac{g_{m2}}{g_{m1}}\right)^2\cdot\left(\frac{\bar{e}_{n2}}{\bar{e}_{n1}}\right)^2}{\left(1+\frac{g_{m2}}{g_{m1}}\right)^2}\right)$$

eq. 13

Noise can be reduced.

Let us examine: Reduction of noise can be achieved by handling the gm´s (eq.2.13). In case of the signal, it is wise that the gm of the input is dominated by the required bandwidth. But we can enlarge the current through the transistors by decrease the load transistor. The consequence is to enlarge the length of the load transistor. L2 > L1 and the area is proportional to the square root of the relation L2 versus L1.

$$gm=\frac{\partial I_D}{\partial V_{GS}}$$

eq. 14

In case of the symmetrical inverter (see illustration 4) , noise can be determined as shown in eq. 2.13. noise reduction is just possible by reduction of area.

There is an open question left:

Where do the different noise voltages come from?

The answer is: some from the basic noise theory, some is presented above, others may be visited in the Internet or from measurements. Measurements can be derived by observing the noisy floor of the circuit of interest. Picking these values and calculating a probability distribution is a possible way. Another way is to have some confidence on network simulators, some of them contribute to noise analysis.

Final remark: ***noise is the most wanted figure!!***

The exercises will show the HOW-TO of noise modelling in MATLAB-SIMULINK.

Literature around noise

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[4] http://en.wikipedia.org/wiki/Shot_noise

[5] http://en.wikipedia.org/wiki/Generation-recombination_noise

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[18] Kwok K. Hung et. Al, “A unified model for the flicker noise in metal-oxid-semiconductor field-effect transistors”, IEEE Trans El. Dev. Vol.37, No.3, March 1990

[19] Kwok K. Hung et al., “A physics-based MOSFET noise model for circuit simulators”, IEEE Trans El.Dev. Vol. 37, No. 5, May 1990

Have a look to the internet: search for “electronic noise, electrical noise, noise in MOS structures, low-frequency noise, noise in amplifiers”